

Double Spin Asymmetries through QCD Instantons

Yachao Qian¹ and Ismail Zahed¹

¹*Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794-3800.*

We revisit the large instanton contribution to the gluon Pauli form factor of the constituent quark noted by Kochelev. We check that it contributes sizably to the single spin asymmetry in polarized $p_{\uparrow}p \rightarrow \pi X$. We use it to predict a large double spin asymmetry in doubly polarized $p_{\uparrow}p_{\uparrow} \rightarrow \pi\pi X$.

I. INTRODUCTION

The QCD vacuum is dominated by large instanton and anti-instanton fluctuations in the infrared, that are largely responsible for the spontaneous breaking of chiral symmetry and the anomalously large η' mass [1, 2]. QCD instantons may contribute substantially to small angle hadron-hadron scattering [3–7] and possibly gluon saturation at HERA [8, 9], as evidenced by recent lattice investigations [10, 11].

A number of semi-inclusive DIS experiments carried by the CLAS and HERMES collaborations [12–15], and more recently with polarized protons on protons by the STAR and PHENIX collaborations [16–18], have revealed large spin asymmetries in polarized lepton-hadron and hadron-hadron collisions at collider energies. These effects are triggered by T-odd contributions in the scattering amplitude.

Perturbative QCD does not support the T-odd contributions, which are usually parametrized in the initial state (Sivers effect) [19, 20] or the final state (Collins effect) [21, 22]. Non-perturbative QCD with instantons allow for large spin asymmetries as discussed by Kochelev and others [23–26]. In [23] a particularly large Pauli form factor was noted, with an important contribution to the Single Spin Asymmetry (SSA) in polarized proton on proton scattering. In this note we would like to point out that it contributes significantly to doubly polarized proton on proton scattering.

The organization of the paper is as follows: In section II we review the emergence of a large Pauli form factor on a constituent quark in the QCD vacuum. In section III we assess its effect on the transverse SSA in $p_{\uparrow}p \rightarrow \pi^{\pm,0}X$ following a recent argument in [27]. The effect is comparable in magnitude to the one discussed in [25, 26]. In section IV we show that it gives a substantial contribution to the Double Spin Asymmetry (DSA) in semi-inclusive $p_{\uparrow}p_{\uparrow} \rightarrow \pi\pi X$. Our conclusions follow in section V.

II. EFFECTIVE PAULI FORM FACTOR

The QCD vacuum is a random ensemble of instantons and anti-instantons interacting via the exchange of perturbative gluons and quasi-zero modes of light quarks and anti-quarks. In the dilute instanton approximation, a typical effective vertex with quarks and gluons attached to an instanton is shown in Fig. 1. The corresponding effective vertex is given by [28–30],

$$\mathcal{L} = \int \prod_q \left[m_q \rho - 2\pi^2 \rho^3 \bar{q}_R \left(1 + \frac{i}{4} \tau^a \bar{\eta}_{\mu\nu}^a \sigma_{\mu\nu} \right) q_L \right] \exp \left(-\frac{2\pi^2}{g_s} \rho^2 \bar{\eta}_{\gamma\delta}^b G_{\gamma\delta}^b F_g(\rho Q) \right) d_0(\rho) \frac{d\rho}{\rho^5} d\bar{\sigma} + (L \leftrightarrow R) \quad (1)$$

where $d\bar{\sigma}$ is the integration over the instanton orientation in color space and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$. The incoming and outgoing quarks have small momenta p ($\rho p \ll 1$) and Q is the momentum transferred by the inserted gluon with a form-factor

$$F_g(x) \equiv \frac{4}{x^2} - 2K_2(x) \xrightarrow{x \rightarrow 0} 1 \quad (2)$$

By expanding Eq. 1 to leading order in the inserted gluon field of $G_{\gamma\delta}^b$ and integrating over the color indices, we obtain

$$\frac{i}{g_s} F_g(\rho Q) \int \pi^4 \rho^4 \frac{\bar{q}_R t^a \sigma_{\mu\nu} q_L}{m_q^*} G_{\mu\nu}^a \times \left(\prod_q (\rho m_q^*) d_0(\rho) \frac{d\rho}{\rho^5} \right) = \frac{i}{g_s} F_g(\rho Q) \int d\rho \pi^4 \rho^4 n(\rho) \frac{\bar{q}_R t^a \sigma_{\mu\nu} q_L}{m_q^*} G_{\mu\nu}^a \quad (3)$$

where $n(\rho)$ is the effective instanton density and m_q^* is the effective quark mass. In the dilute instanton approximation [31]

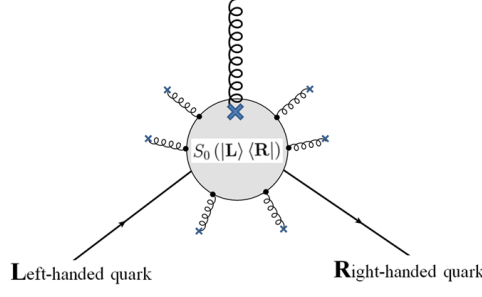


FIG. 1: Effective Quark-Gluon vertex in the instanton vacuum.

$$n(\rho) = n_I \delta(\rho - \rho_c) \quad (4)$$

where ρ_c is the average size of the instanton. Hence the induced instanton effective quark-gluon vertex

$$\frac{i}{g_s} F_g(\rho Q) \pi^4 (n_I \rho_c^4) \frac{\bar{q}_R t^a \sigma_{\mu\nu} q_L}{m_q^*} G_{\mu\nu}^a \quad (5)$$

as illustrated in Fig. 1. In momentum space, the effective vertex is M_μ^a and reads

$$M_\mu^a = \gamma_\mu t^a - \frac{2}{g_s^2} F_g(\rho Q) \pi^4 (n_I \rho_c^4) \frac{t^a \sigma_{\mu\nu} q^\nu}{m_q^*} \quad (6)$$

after analytical continuation to Minkowski Space. Eq. 5 yields an anomalously large Quark Chromomagnetic Moment [30]

$$\mu_a = -\frac{2n_I \pi^4 \rho_c^4}{g_s^2} \quad (7)$$

III. SINGLE SPIN ASYMMETRIES

A. SSA: Estimate

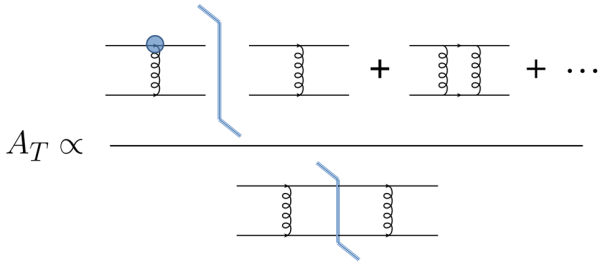


FIG. 2: Schematically diagrammatic contributions to the SSA through the Pauli Form factor [27]

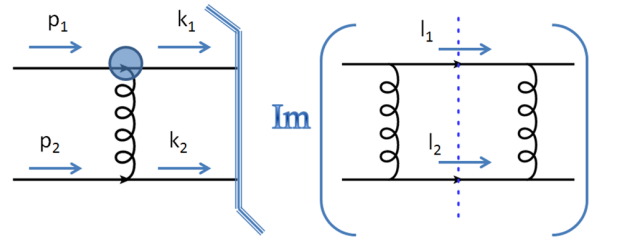


FIG. 3: Leading diagrammatic contribution to the SSA through the Pauli form factor.

To calibrate the effects of Eq. 6 on the double spin asymmetries, we revisit its contribution to the SSA on semi-inclusive and polarized $p_\uparrow p \rightarrow \pi^{\pm,0} X$ experiments. This effect was recently discussed in [27], so we will be brief. In going through an instanton, the chirality of the light quark can be flipped. Using the Pauli form factor discussed in Sec-II, the SSA follows from the diagrams of Fig. 2. As noted in [27], the leading diagram contributing to the SSA

is displayed in Fig. 3. Note that Fig. 3 is of the same order in g_s as the zeroth order diagram in Fig. 2, since the chirality-flip effective vertex (Eq. 6) is semi-classical and of order $1/g_s^2$. The zeroth order differential cross section reads

$$d^{(0)}\sigma \sim \frac{64g_s^4}{|p_1 - k_1|^4} [(k_1 \cdot p_2)(k_2 \cdot p_1) + (k_1 \cdot k_2)(p_1 \cdot p_2)] \quad (8)$$

The first order differential cross section for the chirality flip reads [32]

$$d^{(1)}\sigma \sim i \frac{g_s^6}{(k_1 - p_1)^2} \frac{1}{16\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{\mu^{2\epsilon}}{s^\epsilon} \int_0^1 dy [y(1-y)]^{-\epsilon} \int_0^{2\pi} \frac{d\phi_l}{2\pi} \frac{1}{(l_1 - k_1)^2} \frac{1}{(p_1 - l_1)^2} \mathcal{G}(\Omega) \quad (9)$$

where $y = (1 + \cos \theta_l)/2$, $\pm \theta_l$ is the longitudinal angle of $l_{1/2}$ and

$$\mathcal{G}(\Omega) \equiv \text{tr}[(M_\mu^a)^{(1)} \not{p}_1 \gamma_5 \not{s} \gamma_\nu t^b \not{l}_1 \gamma_\rho t^c \not{k}_1] \text{tr}[\gamma^\mu t_a \not{p}_2 \gamma^\nu t_b \not{l}_2 \gamma^\rho t_c \not{k}_2] \quad (10)$$

From Sec-II

$$(M_\mu^a)^{(1)} = -\frac{t^a}{g_s^2} \frac{F_g(\rho Q) \pi^4 (n_I \rho_c^4)}{m_q^*} [\gamma_\mu (\not{k}_1 - \not{p}_1) + (\not{p}_1 - \not{k}_1) \gamma_\mu] \quad (11)$$

To simplify the analysis and compare to existing semi-inclusive data, we use the kinematics

$$\begin{aligned} p_{1/2} &= \frac{\sqrt{\tilde{s}}}{2} (1, 0, 0, \pm 1) \\ k_{1/2} &= \frac{\sqrt{\tilde{s}}}{2} (1, \pm \sin \theta \sin \phi, \pm \sin \theta \cos \phi, \pm \cos \theta) \\ s &= (0, 0, s^\perp, 0) \end{aligned} \quad (12)$$

where $\sqrt{\tilde{s}}$ is the total energy of the colliding "partons". It is simple to show that $d^{(1)}\sigma \sim \vec{k}_1 \cdot (\vec{p}_1 \times \vec{s}) \sim \sqrt{\tilde{s}} s^\perp k_1^\perp \sin \phi$, which results in SSA. For simplicity, we calculate the first differential cross section $d^{(1)}\sigma$ with $\phi = \pi/2$, where the transverse momentum of the outgoing particle lines along the x axis. Straightforward algebra yields

$$\begin{aligned} d^{(1)}\sigma &\sim s^\perp k_1^\perp \frac{2g_s^4}{3\pi} \frac{\Gamma(-\epsilon)}{\Gamma(2-2\epsilon)\Gamma(1-\epsilon)} \csc^2(\theta) (4\pi)^\epsilon \frac{\mu^{2\epsilon}}{s^\epsilon} \frac{F_g(\rho Q) \pi^4 (n_I \rho_c^4)}{m_q^*} \\ &\times [25\epsilon - 12 + \cos \theta (\epsilon(9 + 2\epsilon) - 4) {}_2F_1(1, 1 - \epsilon, 1 - 2\epsilon, \sec^2 \frac{\theta}{2}) + \epsilon(1 - \cos \theta) {}_2F_1(2, 1 - \epsilon, 1 - 2\epsilon, \sec^2 \frac{\theta}{2})] \end{aligned} \quad (13)$$

where ${}_2F(a, b, c; y)$ is a hypergeometric function. We note that $|{}_2F_1(1, 1, 1; y)|$ is much larger than $|{}_2F_1^{(0,1,0,0)}(1, 1, 1; y)|$ and $|{}_2F_1^{(0,0,1,0)}(1, 1, 1; y)|$ for $y \sim 1$. Therefore

$$d^{(1)}\sigma \sim s^\perp k_1^\perp \frac{2g_s^4}{3\pi} \frac{F_g(\rho Q) \pi^4 (n_I \rho_c^4)}{m_q^*} \csc^4(\frac{\theta}{2}) (3 + \cos \theta) \left(-\frac{1}{\epsilon} + 2\gamma_E + \ln(\frac{\tilde{s}}{4\pi\mu^2}) \right) \quad (14)$$

The divergence in (14) stems from the exchange of soft gluons in the box diagram. In [27] it was regulated using a constituent gluon mass m_g . For $\theta_l \sim 0$, \vec{l}_1 is parallel to \vec{p}_1 , and this collinear divergence could be regulated by restricting $-(l_1 - p_1)^2 > m_g^2$ or equivalently setting $y_{\max} \sim 1 - c m_g^2/\tilde{s}$ with c an arbitrary constant of order 1. This regularization amounts to the substitution

$$\int_0^1 dy \longrightarrow \left(\int_0^{\frac{1+\cos \theta}{2} - c \frac{m_g^2}{\tilde{s}}} + \int_{\frac{1+\cos \theta}{2} + c \frac{m_g^2}{\tilde{s}}}^1 \right) dy \quad (15)$$

in Eq. 9, where we have also regulated the collinear divergence when \vec{l}_1 is parallel to \vec{k}_1 . Thus

$$\left(-\frac{1}{\epsilon} + 2\gamma_E + \ln\left(\frac{\tilde{s}}{4\pi\mu^2}\right)\right) \longrightarrow \ln\left(c\frac{\tilde{s}}{m_g^2}\right) + \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right) \quad (16)$$

The regulated SSA is now given by

$$A_T^{\sin\phi} \approx \frac{d^{(1)}\sigma}{d^{(0)}\sigma} = s^\perp k_1^\perp \frac{F_g(\rho Q) \pi^3 (n_I \rho_c^4)}{m_q^*} \frac{(3 + \cos\theta)}{6(5 + 2\cos\theta + \cos^2\theta)} \left[\ln\left(c\frac{\tilde{s}}{m_g^2}\right) + \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right) \right] \quad (17)$$

where the zeroth order cross section in Eq. 8 is used for normalization.

B. SSA: Experiment

To compare with the semi-inclusive data on $p_T p \rightarrow \pi X$, we set $s^\perp u(x, Q^2) = \Delta_s u(x, Q^2)$ and $s^\perp d(x, Q^2) = \Delta_s d(x, Q^2)$, with $\Delta_s u(x, Q^2)$ and $\Delta_s d(x, Q^2)$ as the spin polarized distribution functions of the valence up-quarks and valence down-quarks in the proton respectively. For forward π^+ , π^- and π^0 productions, the SSAs are

$$A_T^{\sin\phi}(\pi^+) = \left(n_I \frac{\rho^4}{m_q^*}\right) k^\perp \frac{\Delta_s u(x_1, Q^2)}{u(x_1, Q^2)} \pi^3 F_g(\rho Q) \frac{(3 + \cos\theta)}{6(5 + 2\cos\theta + \cos^2\theta)} \left[\ln\left(c\frac{\tilde{s}}{m_g^2}\right) + \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right) \right] \quad (18)$$

$$A_T^{\sin\phi}(\pi^-) = \left(n_I \frac{\rho^4}{m_q^*}\right) k^\perp \frac{\Delta_s d(x_1, Q^2)}{d(x_1, Q^2)} \pi^3 F_g(\rho Q) \frac{(3 + \cos\theta)}{6(5 + 2\cos\theta + \cos^2\theta)} \left[\ln\left(c\frac{\tilde{s}}{m_g^2}\right) + \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right) \right] \quad (19)$$

$$A_T^{\sin\phi}(\pi^0) = \left(n_I \frac{\rho^4}{m_q^*}\right) k^\perp \frac{\Delta_s u(x_1, Q^2) + \Delta_s d(x_1, Q^2)}{u(x_1, Q^2) + d(x_1, Q^2)} \pi^3 F_g(\rho Q) \frac{(3 + \cos\theta)}{6(5 + 2\cos\theta + \cos^2\theta)} \left[\ln\left(c\frac{\tilde{s}}{m_g^2}\right) + \ln\left(\frac{1 - \cos\theta}{1 + \cos\theta}\right) \right] \quad (20)$$

According to [33, 34]

$$\begin{aligned} \frac{\Delta_s u(x, Q^2)}{u(x, Q^2)} &= 0.959 - 0.588(1 - x^{1.048}) \\ \frac{\Delta_s d(x, Q^2)}{d(x, Q^2)} &= -0.773 + 0.478(1 - x^{1.243}) \\ \frac{u(x, Q^2)}{d(x, Q^2)} &= 0.624(1 - x) \end{aligned} \quad (21)$$

These results can be compared to the experimental measurements in [35]. For simplicity, we assume the same fraction for each proton $\langle x_1 \rangle = \langle x_2 \rangle = \langle x \rangle$, and $\langle k^\perp \rangle \approx \langle K_\perp \rangle$ is the transverse momentum of the outgoing pion. We then have $\sqrt{s} \langle x \rangle \langle \sin\theta \rangle = 2 \langle K_\perp \rangle$ and $\langle x \rangle \langle \cos\theta \rangle = \langle x_F \rangle$. For large \sqrt{s} , we also have $\langle Q \rangle \approx \langle K_\perp \rangle \sqrt{\langle x \rangle / \langle x_F \rangle}$. We set $c = 2$ and $\langle K_\perp \rangle = 2\text{GeV}$ for the outgoing pions. $n_I \approx 1/\text{fm}^4$ is the effective instanton density, $\rho \approx 1/3\text{fm}$ the typical instanton size and $m_q^* \approx 300\text{MeV}$ the constitutive quark mass in the instanton vacuum. $m_g \approx 420\text{MeV}$ is the effective gluon mass in the instanton vacuum[36]. In Fig. 4 (left) we display the results (18-20) as a function of the parton fraction x_F for both the charged and uncharged pions at $\sqrt{s} = 19.4\text{GeV}$ [35].

Instead of isolating the collinear divergence, we can also numerically compute Eq. 9 with a massive gluon propagator as was discussed also in [27]

$$d^{(1)}\sigma \sim i \frac{g_s^6}{(k_1 - p_1)^2 - m_g^2} \frac{1}{16\pi} \int_0^1 dy \int_0^{2\pi} \frac{d\phi_l}{2\pi} \frac{1}{(l_1 - k_1)^2 - m_g^2} \frac{1}{(p_1 - l_1)^2 - m_g^2} \mathcal{G}(\Omega) \quad (22)$$

The numerical results are displayed in Fig. 4 (right). Both regularizations lead about similar results. In sum, the anomalous Pauli form factor can reproduce the correct magnitude of the observed SSA in polarized $p_T p \rightarrow \pi X$ for reasonable vacuum parameters. The contribution is comparable in magnitude to the one recently discussed in [26] (see Fig. 11 in p.7) using instead the standard Dirac form factor but changes in the instanton distribution due to the polarized proton.

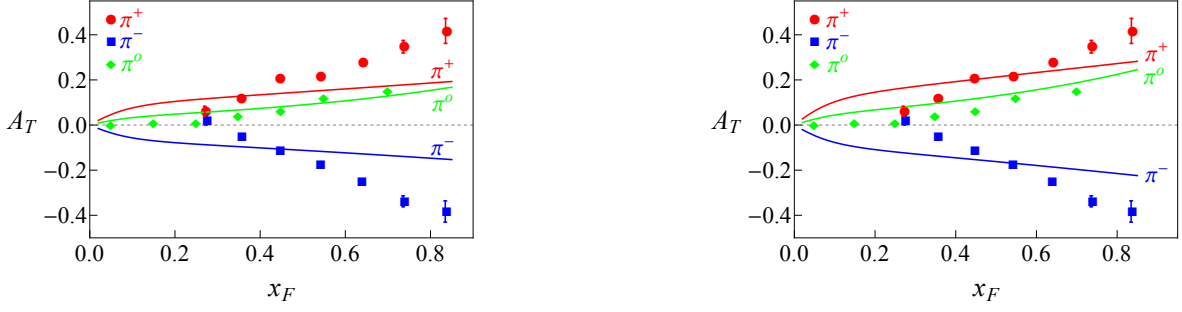


FIG. 4: x_F dependent SSA in $p_T p \rightarrow \pi X$ collisions at $\sqrt{s} = 19.4 \text{ GeV}$ [35]. The solid lines are the analytical results in Eq. 18-Eq. 20 with $c = 2$ (left) and Eq. 22 (right).

IV. DOUBLE SPIN ASYMMETRIES IN DIJET PRODUCTIONS

A. DSA: Estimate

The same Pauli form factor and vacuum parameters can be used to assess the role of the QCD instantons on doubly polarized and semi-inclusive $p_T p \rightarrow \pi \pi X$ processes. The Double Spin Asymmetry (DSA) is defined as

$$A_{\text{DS}} = \frac{\sigma^{\uparrow\uparrow+\downarrow\downarrow} - \sigma^{\downarrow\downarrow+\uparrow\uparrow}}{\sigma^{\uparrow\uparrow+\downarrow\downarrow} + \sigma^{\downarrow\downarrow+\uparrow\uparrow}} \quad (23)$$

with the proton beam polarized along the transverse direction. The valence quark from the polarized proton P_1 exchanges one gluon with the valence quark from the polarized proton P_2 as shown in Fig. 5. At large \sqrt{s} , Fig. 5-(a) is dominant in forward pion production and Fig. 5-(b) is dominant in backward pion production. For Fig. 5-(a), the differential cross section reads

$$d\sigma \sim \frac{g_s^4}{|p_1 - k_1|^4} \sum_{\text{color}} \text{tr} [M_\mu^a \not{p}_1 (1 + \gamma_5 \not{s}_1) \gamma_0 (M_\nu^b)^\dagger \gamma_0 \not{k}_1] \text{tr} [M_\mu^a \not{p}_2 (1 + \gamma_5 \not{s}_2) \gamma_0 (M_\nu^b)^\dagger \gamma_0 \not{k}_2] \quad (24)$$

Using the anomalous Pauli form factor (6), the contribution to the DSA follows from simple algebra

$$\begin{aligned} d^{(2)}\sigma \sim & \frac{256}{|p_1 - k_1|^4} \left(\frac{F_g(\rho Q) \pi^4 n_I \rho_c^4}{m_q^*} \right)^2 [(k_1 \cdot s_1)(k_1 \cdot s_2)(k_2 \cdot p_1)(k_2 \cdot p_2) - (k_1 \cdot p_1)(k_1 \cdot s_2)(k_2 \cdot p_2)(k_2 \cdot s_1) \\ & - (k_1 \cdot s_1)(k_1 \cdot s_2)(k_2 \cdot p_2)(p_1 \cdot p_2) + (k_1 \cdot k_2)(k_1 \cdot p_1)(k_2 \cdot p_2)(s_1 \cdot s_2) - (k_1 \cdot p_1)(k_1 \cdot p_2)(k_2 \cdot p_2)(s_1 \cdot s_2) \\ & - (k_1 \cdot p_1)(k_2 \cdot p_1)(k_2 \cdot p_2)(s_1 \cdot s_2) + (k_1 \cdot p_1)(k_2 \cdot p_2)(p_1 \cdot p_2)(s_1 \cdot s_2) - (k_1 \cdot p_2)(k_1 \cdot s_1)(k_2 \cdot p_1)(k_2 \cdot s_2) \\ & + (k_1 \cdot p_1)(k_1 \cdot p_2)(k_2 \cdot s_1)(k_2 \cdot s_2) + (k_1 \cdot k_2)(k_1 \cdot s_1)(k_2 \cdot s_2)(p_1 \cdot p_2) - (k_1 \cdot p_1)(k_2 \cdot s_1)(k_2 \cdot s_2)(p_1 \cdot p_2)] \end{aligned} \quad (25)$$

after using the identity

$$\begin{aligned} & \text{tr} [(\gamma_\mu \not{q} - \not{q} \gamma_\mu) \not{p} \gamma_5 \not{s} \gamma_\nu \not{k}] + \text{tr} [\gamma_\mu \not{p} \gamma_5 \not{s} (\not{q} \gamma_\nu - \gamma_\nu \not{q}) \not{k}] \\ & = \text{tr} [(\gamma_\mu \not{k} + \not{p} \gamma_\mu) \not{p} \gamma_5 \not{s} \gamma_\nu \not{k}] + \text{tr} [\gamma_\mu \not{p} \gamma_5 \not{s} (\not{k} \gamma_\nu + \gamma_\nu \not{k}) \not{p}] \\ & = 8i [p_\mu \epsilon(\nu, k, p, s) - p_\nu \epsilon(\mu, k, p, s) + (k \cdot p) \epsilon(\mu, \nu, k, s) - (k \cdot s) \epsilon(\mu, \nu, k, p)] \end{aligned} \quad (26)$$

where we used $q = k - p$ and $p \cdot s = 0$ because the protons are transversely polarized.

For a simple empirical application of (25) we adopt the simple kinematical set up in Eq. 12. Obtain

$$d^{(2)}\sigma \sim -\frac{4}{|p_1 - k_1|^4} \left(\frac{F_g(\rho Q) \pi^4 n_I \rho_c^4}{m_q^*} \right)^2 \tilde{s}^3 s_1^\perp s_2^\perp (1 - \cos \theta)^2 [4 + \cos(\theta - 2\phi) + 2 \cos(2\phi) + \cos(\theta + 2\phi)] \quad (27)$$

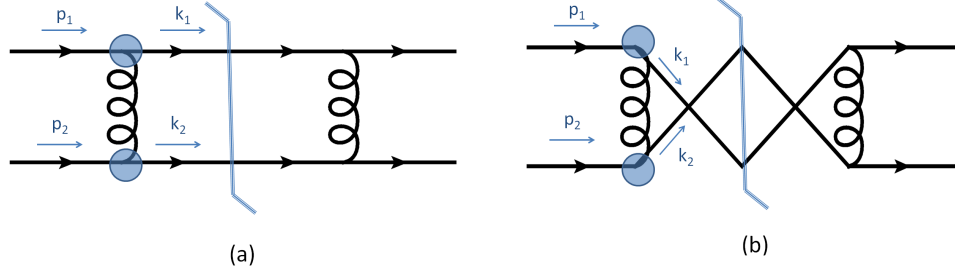


FIG. 5: The valence quark in polarized proton p_1 exchange one gluon with the valence quark in the polarized proton p_2 .

After adding the contribution of Fig. 5-(a) and Fig. 5-(b), and averaging over the transverse direction ϕ , we finally obtain

$$\frac{d^{(2)}\sigma}{d^{(0)}\sigma} \sim -4s_1^\perp s_2^\perp \left(\frac{\pi^4 n_I \rho_c^4}{m_q^* g_s^2} \right)^2 \frac{F_g^2 [\rho \sqrt{\frac{\tilde{s}(1-\cos\theta)}{2}}] \tilde{s} + F_g^2 [\rho \sqrt{\frac{\tilde{s}(1+\cos\theta)}{2}}] \tilde{s}}{\frac{5+2\cos\theta+\cos^2\theta}{(1-\cos\theta)^2} + \frac{5-2\cos\theta+\cos^2\theta}{(1+\cos\theta)^2}} \quad (28)$$

B. DSA: Results

Our DSA results can now be compared to future experiments at collider energies. Specifically, our DSA for dijet productions are

$$A_{\pi^+\pi^+} = -\frac{1}{8} \frac{\Delta_s u(x_1, Q^2)}{u(x_1, Q^2)} \frac{\Delta_s u(x_2, Q^2)}{u(x_2, Q^2)} \left(\frac{\pi^3 n_I \rho_c^4}{m_q^* \alpha_s} \right)^2 \frac{F_g^2 [\rho \sqrt{\frac{\tilde{s}(1-\cos\theta)}{2}}] \tilde{s} + F_g^2 [\rho \sqrt{\frac{\tilde{s}(1+\cos\theta)}{2}}] \tilde{s}}{(5 + 10 \cos^2 \theta + \cos^4 \theta) \csc^4 \theta} \quad (29)$$

$$A_{\pi^-\pi^-} = -\frac{1}{8} \frac{\Delta_s d(x_1, Q^2)}{d(x_1, Q^2)} \frac{\Delta_s d(x_2, Q^2)}{d(x_2, Q^2)} \left(\frac{\pi^3 n_I \rho_c^4}{m_q^* \alpha_s} \right)^2 \frac{F_g^2 [\rho \sqrt{\frac{\tilde{s}(1-\cos\theta)}{2}}] \tilde{s} + F_g^2 [\rho \sqrt{\frac{\tilde{s}(1+\cos\theta)}{2}}] \tilde{s}}{(5 + 10 \cos^2 \theta + \cos^4 \theta) \csc^4 \theta} \quad (30)$$

$$A_{\pi^+\pi^-} = -\frac{1}{8} \frac{\Delta_s u(x_1, Q^2) \Delta_s d(x_2, Q^2) + \Delta_s d(x_1, Q^2) \Delta_s u(x_2, Q^2)}{u(x_1, Q^2) d(x_2, Q^2) + d(x_1, Q^2) u(x_2, Q^2)} \left(\frac{\pi^3 n_I \rho_c^4}{m_q^* \alpha_s} \right)^2 \times \frac{F_g^2 [\rho \sqrt{\frac{\tilde{s}(1-\cos\theta)}{2}}] \tilde{s} + F_g^2 [\rho \sqrt{\frac{\tilde{s}(1+\cos\theta)}{2}}] \tilde{s}}{(5 + 10 \cos^2 \theta + \cos^4 \theta) \csc^4 \theta} \quad (31)$$

We will assume that each parton carries one third of the momentum of the proton $\langle x_1 \rangle = \langle x_2 \rangle = 1/3$, so that $\sqrt{\tilde{s}} = \sqrt{s}/3$, where \sqrt{s} is the total energy of the colliding protons. We will use the value of α_s from [37]. The instanton size is set to $1/3\text{fm}$, density $n_I = 1/\text{fm}^4$ and $m_q^* = 300\text{MeV}$ as for the SSA reviewed above. Our predictions for charged dijet production in semi-inclusive DSA are displayed in Fig. 6. We note that at $\sqrt{s} \rightarrow \infty$, the Double Spin Asymmetries in (Eq. 29 , Eq. 30, and Eq. 31) vanish.

V. CONCLUSIONS

QCD instantons provide a natural mechanism for large spin asymmetries in polarized dilepton and hadron scattering at collider energies. A simple mechanism for these large spin asymmetries was noted by Kochelev [23] in the form of a large Pauli form factor for a constituent quark whether through photon exchange or gluon exchange. A simple estimate of the SSA in $p_\uparrow p \rightarrow \pi X$ production compares fairly to the measured charged asymmetries in [35] both in sign and magnitude, using the instanton vacuum parameters. We have argued that the same anomalously large Pauli form factor yields substantial DSA in $p_\uparrow p_\uparrow \rightarrow \pi\pi X$. We welcome future measurements of these asymmetries at collider facilities.

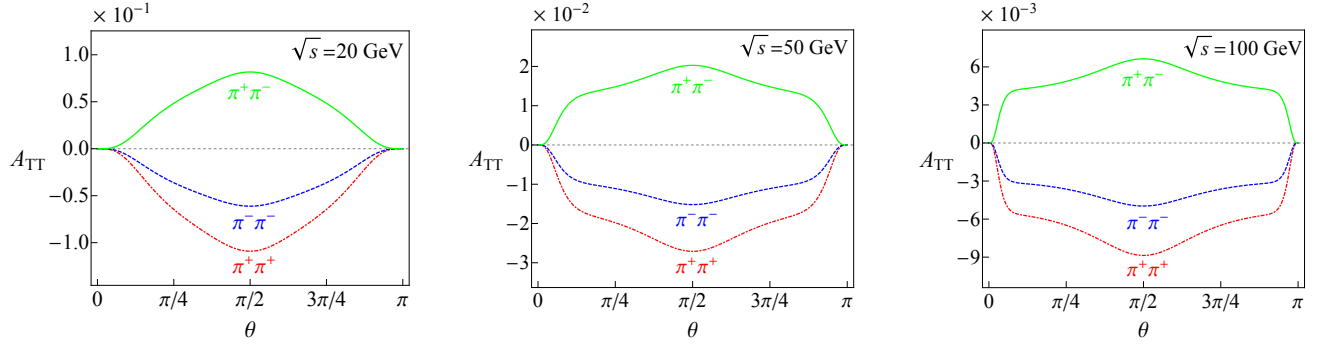


FIG. 6: Dotdashed line is the Double Spin Asymmetry of $\pi^+\pi^+$ productions (Eq. 29). Dashed line is the Double Spin Asymmetry of $\pi^-\pi^-$ productions (Eq. 30). Solid line is the Double Spin Asymmetry of $\pi^+\pi^-$ productions (Eq. 31).

VI. ACKNOWLEDGEMENTS

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